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SOME DYNAMICAL ASPECTS OF STATIC STABILITY

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ABSTRACT

The relation between static stability of the atmosphere and wave length of flow patterns is investigated for two sets of data: (1) Northern Hemispheric charts for a 10-day period, December 26, 1952–January 4, 1953 and (2) United States charts for February 15–June 1, 1953. Results for the first set of data appear to indicate a dynamically significant relation between the wave length at 500 mb. and the ratio of square root of mean stability to mean Coriolis parameter. Over the United States, this relationship does not appear to be sufficiently strong to be of direct applicability in forecasting.

INTRODUCTION

The purpose of this investigation is to study the relation between static stability and the dimensions of the flow patterns. Such an investigation is suggested by the occurrence of the important ratio, square root of the stability over Coriolis parameter, in the theory of certain atmospheric models in which vertical motion is allowed. For example, in the so-called 2½ dimensional model described by Eliassen [1] and related models discussed by Eady [2, 3] and others, the wave length L_x corresponding to maximal growth rate of a disturbance is found to be expressed by

$$L_x = K S_1^{1/2} f^{-1} \qquad (1)$$

where K is a constant (dimensional), f is the Coriolis parameter, and S_1, the static stability in the infinitesimal layer, is given by

$$S_1 = g \frac{\partial}{\partial z} (\ln \theta) \qquad (2)$$

where θ is the potential temperature. Instead of local lapse rates, it is convenient to measure the mean stability, S, in a layer of appreciable thickness between two isobaric layers. It can be shown that for all practical purposes one may take S_1 as proportional to S if

$$S = T_t - \theta_b \qquad (3)$$

where T_t is the temperature at the top of the layer considered and θ_b is the temperature which a parcel of air at the base of the layer would attain if it were lifted dry-adiabatically to the top of the layer. For example the layer 850–500 mb. might be studied using the 850-mb. temperature to compute θ_b and the 500-mb. temperature as T_t.

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[2] Research accomplished while on temporary duty in the Weather Forecasting Research Center.

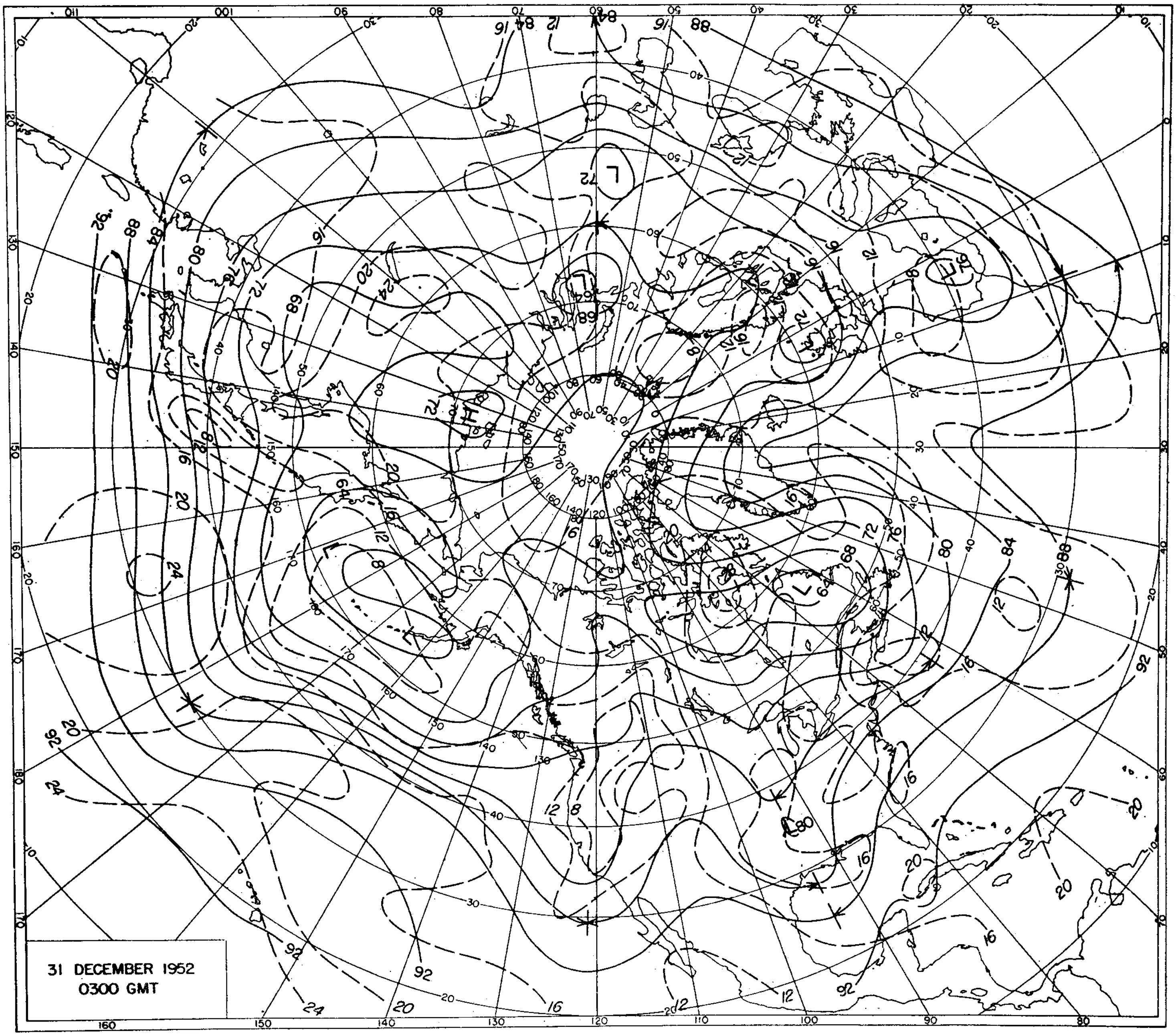

FIGURE 1.—Stability lines (broken, in °C) and contour lines (solid) for 500-mb. surface, 0300 GMT, December 31, 1952.

SELECTION OF DATA

The first set of data studied was obtained from maps covering a 10-day period which was chosen at random. This period, December 26, 1952, to January 4, 1953, was chosen principally because carefully analyzed Northern Hemisphere charts, produced at the Weather Forecasting Research Center, were available for the period. These charts were used to compile data on the hemispherical aspects of stability. In addition, the period February 15 to June 1, 1953, was chosen for a repetition of the investigation, using only the synoptic data for the part of North America which has a dense network of sounding stations. The first set of data contained 97 individual waves whose lengths were correlated with the stability, and the second set of data contained 51 such waves.

STABILITY AND WAVES

Waves were selected on the 500-mb. chart and were identified by a defining contour which was the portion of the actual 500-mb. contour whose physical form best represented the wave selected. To avoid the inclusion of a wave more than once, to eliminate the effects of ground inversions, and otherwise to facilitate the investigation, the following criteria for selection of cases were developed:

(1) The axes of the wave troughs and wedges must be oriented along meridians or nearly so.

(2) The amplitude of the wave must be small so that it is permissible to use a mean value of the Coriolis parameter in the computation of $S^{1/2}f^{-1}$.

(3) Only one defining contour may be used for any given wave, and a situation in which a contour defines more than one wave is preferable (i. e., wherever possible two adjacent waves were included in the data).

(4) Semipermanent long waves are to be excluded since their amplitudes are usually large and their frequency such that they unduly weight the statistics. Further, long waves are theoretically controlled by distant and broad-scale effects.

(5) Waves are not to be selected from regions where it is impossible to correct for the effect of surface inversions on the 850-mb. temperature. (It is clear from equation (3) that if this temperature is not representative the stability value computed from it cannot be either.)

Some examples of the waves selected are indicated by the arrows in figure 1.

Using equation (3) and considering the layer 850 mb. to 500 mb., stability values were computed at each of about 40 points (the exact number depending on the length of the particular wave) within an area of 5° of latitude either side of the defining contour. Actually for the basic 10-day period, hemispheric charts of stability were constructed. (See, for example, fig. 1.) Furthermore, in mountainous portions of North America where the 850-mb. temperature was influenced by the surface inversion, the lapse rate near the ground was straightened to provide a representative 850-mb. temperature. All computations were based on the 0300 GMT chart.

The mean meridional distribution of the stability as obtained from the hemispherical stability chart for each day of the 10-day period is shown in figure 2. Appreciable daily fluctuations of this value are indicated. The curves on the right show the mean values of S and $S^{1/2}f^{-1}$ for the whole period. It will be seen that, on the average, there is a minimum of stability, S, at about 45°N., while the parameter $S^{1/2}f^{-1}$ varies but little to the north of this latitude.

Returning now to consideration of individual waves, the mean stability for the wave, denoted by S_m, was obtained by taking the arithmetic mean of the values computed at the points along the wave. The square root of this mean value of the stability was then divided by the value of the Coriolis parameter which corresponded to the mean latitude of the wave to obtain $S_m^{1/2}f_m^{-1}$ for each wave. These values were then tabulated against the wave lengths of the corresponding waves. Finally a correlation coefficient was computed. This process was carried through for each of the two sets of data described above. Correlation coefficients obtained are tabulated in line one of table 1.

It will be noted that for the hemispheric data the correlation between L and $S_m^{1/2}f_m^{-1}$ is quite high (0.82) but

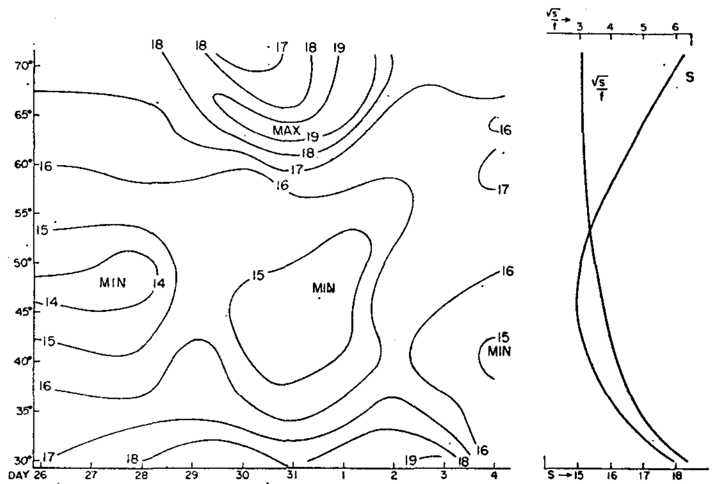

FIGURE 2.—Time sequence of mean meridional profiles of stability. The curves on the right show the mean meridional stability and the mean distribution of the quantity $S^{1/2}f^{-1}$ during the period. S in ° C; $f=2\Omega \sin \phi \times 10^4$ sec.$^{-1}$

that the corresponding value drops to 0.39 for the second set of data taken over a limited area in or near the United States.

TABLE 1.—*Correlations between wave length and stability-Coriolis quantities. L is the wave length measured linearly along the 500-mb. contour, S is stability factor given by equation (3) for the layer indicated, f is the Coriolis parameter, subscript m indicates spatial mean as explained in text, r is correlation coefficient*

Correlated quantities	Hemispheric data	United States data		
	850–500 mb.	850–500 mb.	700–500 mb.	700–300 mb.
	r	r	r	r
$L, S_m^{1/2}f_m^{-1}$	0.82	0.39	0.35	0.38
L, f_m^{-1}	.66	.07		
L, S_m		.36		
$L, (S^{1/2}f^{-1})_m$		.48	.42	.45

An explanation for the sharp decrease in the correlation coefficient may be found in the fact that most of the waves of the second set of data lie in a narrow band between 37° and 42° N. Also, in dealing with waves wholly within the dense network of observations in or near the United States an upper limit of about 60° of longitude was put on the waves. Thus, with the second set of data the investigation was restricted to short waves while the first set of data included longer waves as well. Furthermore, geographic effects were largely averaged out in the first set of data since waves were selected at all longitudes. In the second set of data restricted to a narrow band of longitude these effects were not averaged out. The effects include orographic and land-water influences both of which cause the wave length and stability to vary independently of each other. (In one case a wave produced solely by orographic effects is possible while in the other case stability is significantly changed by non-adiabatic surface effects as well as mechanically driven vertical motions.)

In interpreting the above correlations, the question naturally arises whether there is a purely geometrical relation between the linear wave length L and the latitude ϕ. If λ is the angular wave length and R is the radius of the earth, we may write $L=\lambda R \cos \phi$ or $L=R\Omega f^{-1}(\lambda \sin 2\phi)$ where f, as before, is $2\Omega \sin \phi$. The correlation coefficient between L and f^{-1} depends upon the quantity $\lambda \sin 2\phi$. It should be noticed that in the latitude band from 30° to 60° N., where the majority of the waves occurred, $f^{-1}\times 10^{-4}$ varies continuously between 1.37 sec. and 0.792 sec., while $\sin 2\phi$ varies only from 0.866 at the borders to 1.000 at 45° N. It is conceivable that the general circulation of the selection of waves during the period examined, could have been such that variations in the quantity $\lambda \sin 2\phi$ could introduce a higher correlation than otherwise. However, an analysis of λ as a function of ϕ and of the selection of waves indicates that there is no basis for explaining the correlation coefficients as a result of particular circulation patterns or by the selection of waves. It is also possible that paucity of data in low latitudes leads to an unwarranted extension of high latitude troughs into lower latitudes with a consequent geometrical relationship between the wave length L and $(\sin \phi)^{-1}$. A reconsideration of the original charts indicates that such a systematic southward extension of troughs did not occur unless supported by data in the lower latitudes. It appears therefore that both the variation $\overline{S}$ and f are important.

It is of interest to determine the correlation coefficient between L and f_m^{-1}. The low correlation (see line 2 of table 1) for the second set of data is due principally to the narrow band of latitude in which waves were selected and the consequent small variation in the value of f_m. For the second set of data a correlation coefficient was computed between L and S_m (line 3 of table 1).

Using the second set of data stability computations similar to the ones described above were made for two other layers, viz, 700–500 mb. and 700–300 mb. In the latter layer the stability was corrected at all stations where the tropopause was below the 300-mb. level. Correlation coefficients between L (at 500 mb.) and the stability factors for these two layers are given in the last two columns of table 1.

In all computations described thus far a single value of the Coriolis parameter corresponding to the mean latitude of the wave was used. Next, the ratio $S^{1/2}f^{-1}$ was computed by taking into account the value of f at each individual grid point. The ratios obtained for all points were then arithmetically averaged to give a new ratio which we designate as $(S^{1/2}f^{-1})_m$. This gives, in every case, a better correlation (see last line of table 1).

STABILITY PATTERNS ASSOCIATED WITH WAVES

In an attempt to construct a composite pattern of stability along and across the waves at the 500-mb. level, the stability in the layer 850–500 mb. in each wave was evaluated at troughs, wedges, and inflection lines. The grids used in this evaluation are shown by the dots in figures 3 and 4 while the heavy line symbolizes the defining contour. The north-south interval between dots is 5° of latitude while the east-west interval is a fractional part of the wave length.

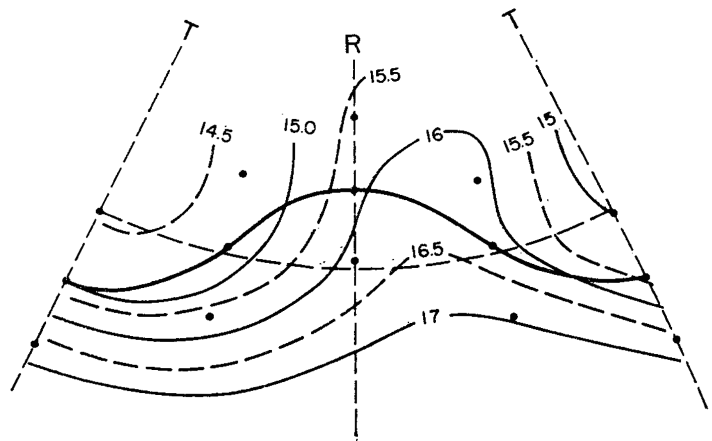

FIGURE 3.—Mean distribution of the stability in the layer from 850 to 500 mb. relative to the mean position of the defining contour (heavy line) at the 500-mb. level. T and R represent trough and ridge lines, respectively.

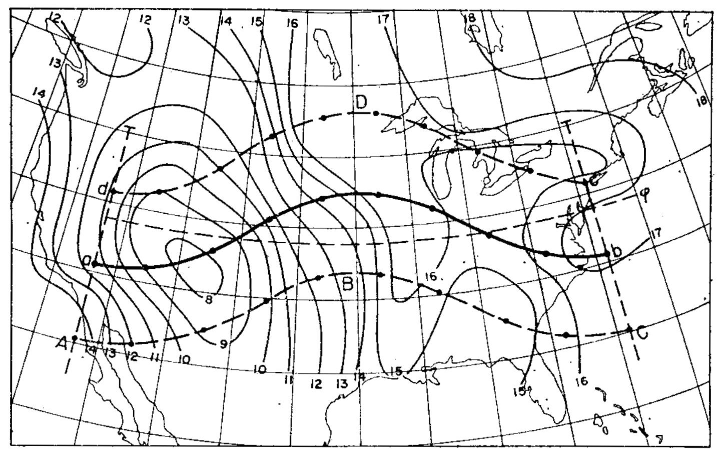

FIGURE 4.—Mean distribution of stability in the layer 850 mb. to 500 mb. for the cases considered. The heavy solid line represents the defining contour and the broken lines the band (A B C c D d A) over which the stability was computed at the grid points (dots).

Figure 3 shows the composite stability pattern for all waves in the first set of data. It will be seen that on the whole the stability is smaller at the troughs than at the ridges. Since data shown in figure 3 refer to the hemisphere as a whole it is believed that the pattern is fairly representative of waves of relatively small amplitude (waves of large amplitude were not included). It will be seen that the stability pattern is displaced somewhat relative to the contour pattern. It is also noted that the distribution of stability along the two trough lines was not exactly the same, because only part of the waves were adjacent and defined by the same contour.

Figure 4 refers to the second set of data in the 850–500-mb layer. This stability pattern contrasts sharply with the pattern obtained from the hemispherical data. The most striking feature is in the much stronger gradients of S when computed over a limited area of North America. This makes the evaluation and interpretation of mean values more questionable. The asymmetry of the distribution is also much greater. The eastern part of the trough is more unstable and the minimum stability is smaller and is displaced approximately ¼ wave length to the east of the western trough—all perhaps due to the influence of the Rockies. There is also a radical difference in stability pattern over the northwestern portion of the trough (i. e., over southeastern Canada). In this area the surface "cold source" produces a local area of high stability.

CONCLUSIONS

The results of the analysis of the hemispheric set of data (which should be largely free of local influences) appear to indicate that there is a dynamically significant relationship between the wave length at the 500-mb. level and the parameter $S_m^{1/2} f_m^{-1}$.

Over continental United States the relationship between the wave length at the 500-mb. level and the quantity $S_m^{1/2} f_m^{-1}$ does not appear to be sufficiently strong to be of direct applicability in forecasting. The weakness of the relationship in the United States may well be due to strong topographic influences peculiar to the area and does not necessarily preclude usefulness of the relationship elsewhere.

For the United States at least, the layer through which the stability is computed does not appear to be critical since shifting the layer did not appreciably change the correlation coefficient.

For the United States data, better results are obtained in every case by taking the appropriate value of f^{-1} for each point in the grid rather than using a mean value of f^{-1} for the entire wave.

In evaluating the static stability, caution must be exercised to obtain representative values. In particular, it is necessary to eliminate the effects of shallow ground inversions.

The results of this investigation should be regarded only as indications of real relationships, and further investigations, dealing with different types of waves, wider spread of amplitudes, and varying synoptic situations would seem highly desirable.

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